

Theoretical and experimental study of wave
propagation in brass musical instruments



Jonathan A Kemp

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Abstract

The concept of input impedance is a very useful representation of the resonance characteristics of an acoustic horn. A large part of this work discusses its theoretical and experimental determination. It is demonstrated that higher modes, waves with a non-uniform pressure distribution on the plane perpendicular to the axis of the instrument, should be used in the theory of wave propagation in musical instruments featuring a flared bell as an improvement on assuming plane wave propagation.

The impedance at the output end of an acoustic horn is known as the radiation impedance. The existing method for the calculation of the multimodal radiation impedance of a cylindrical tube terminated in an infinite baffle is reviewed. New work is then presented for the calculation of the radiation impedance of a rectangular duct terminated in an infinite baffle. An existing method for calculating the input impedance of an acoustic horn of cylindrical cross-section starting from the radiation impedance is utilised. The method is then formulated for horns of rectangular cross-section. Pressure field calculations are also presented.

In acoustic pulse reflectometry an acoustic pulse is directed into the object under test and the sampled reflections analysed to provide the internal profile and the input impedance. It is shown that better agreement is observed between the experimental and theoretical input impedance when higher modes are included in the calculation.

Currently the bore reconstruction analysis assumes plane wave propagation since this provides a simple formula for the frequency independent reflection and transmission coefficients at changes in cross-section in a pipe. The multimodal reflection and transmission coefficients are, however, frequency dependent. A higher-mode method is presented to calculate the reflection of an impulse with the aim of improving the technique's accuracy for horns which feature a large rate of flare at the end. Digital filters designed to represent losses and cancel reflection from the sound source are also shown to increase accuracy and make possible the measurement of longer objects.

Declaration

I declare that this thesis has been composed by me and that the work is my own.

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